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This report summarizes research accomplishments on stochstic flows in networks. The highlight is a new family of probability distributions for describing the numbers of units at the nodes in partially balanced stochastic networks. Such a distribution is a key tool for evaluating the performance and design of a network. Another major accomplishment is the solution of a long-standing problem of finding an expression for the mean time for one unit to move from one sector of a network to another sector. We also developed several models for concurrent movement of units in networks and batch processing at nodes.

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# Project Title: Stochastic Flows in Networks

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# Stochastic Flows in Networks

## 1 Introduction

This report summarizes our research accomplishments during the last year. The theme of our research is the equilibrium or ergodic behavior of stochastic network processes. Such a process describes the movement of discrete units (customers, parts, data packets etc.) in a network of nodes that process the units. Such processes are often called queueing networks. Some archetypal stochastic networks are as follows:

**Computer Networks:** Transactions, data packets or programs move among processors, computers, peripheral equipment or files.

**Flexible Manufacturing Networks:** Parts, tools or material move among a group of work stations and storage areas that machine and store the units for later use or for shipping.

**Telecommunications Networks:** Telephone calls, data packets or messages move among operators or switching stations.

**Maintenance and Logistic Networks:** Reparable parts or equipment needed for the operation of a large system move among locations where they are used, repaired, and stored.

**Distribution Networks:** Goods, orders or trucks move among plants, warehouses or market locations.

**Biological Networks:** Animals, cells, molecules, neurons, etc. move among locations, states or shapes.

The major concern associated with a stochastic network is to describe the probabilistic behavior of the network in terms of the equilibirum or stationary probability distribution of the numbers of units at the nodes. This distribution is used to derive various performance measures of the network such as the expected cost of operating the network or the percentage of time a sector of the network is overloaded. It is also a basic ingredient for the development of mathematical programming algorithms to select optimal network designs and operating rules. Other important network features include the rate of flow of units on the arcs and through the nodes (the throughputs), the time a unit spends in the network, and the time it takes

for a unit to move from one sector of the network to another.

The existing theory of network processes is primarily for Jackson network processes and its relatives. The key features of these processes are:

- The units move one at a time.
- The nodes operate independently.
- The transition rates depend on only local information: the service rate at a node depends on only the number of units at that node and is independent of the rest of the network.
- The routes of units are independent of each other and hence independent of the congestion in the network.
- The equilibrium distribution is of product form.

By its very nature, however, a network is a system of interacting nodes in which the operation of a node and the routing of a unit may depend on what is happening throughout the network. Examples of dependencies are:

- Parallel or synchronous processing at units of several nodes.
- Alternate routing of units to avoid congestion.
- Accelerating or decelerating the processing rate at a node whenever downstream nodes are starved or congested.
- Units are blocked from entering a sector of the network when the sector cannot handle any more units.

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Such dependencies are ominpresent in the networks mentioned above. To model networks with dependenc[illegible] will require a new generation of network processes that will typically have [illegible]e complex, non-product-form equilibrium distributions. We have made significant progress in this regard. The following sections give an overview of the results we obtained last year.

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# 2 Markovian Network Processes: Congestion-Dependent Routing and Processing

Our work on this subject will appear in an article with the title above in the journal *Queueing Systems Theory and Applications*. This is a relatively new journal that is to be affiliated with the Operations Research Society of America; I am one of its associate editors. The cumulative papers on this research project is in the appendix.

Prior to this paper, there was no theory for stochastic network processes for networks that have been dependent nodes or congestion-dependent routing. A major hindrance was that no one knew what type of multivariate equilibrium distribution would be appropriate for the numbers of units at the nodes in such a network. The existing theory with product form distributions and reversible ideas was no help in this regard. In this paper, we introduce a wide class of Markovian network processes with system-dependent transition rates that represent a variety of interactions between nodes. This class contains the Jackson processes and essentially all of its generalizations developed to date that have closed-form expressions for its equilibrium distribution. We discovered a new family of multivariate distributions for these processes. These distributions appear to be rather universal and may apply to other multivariate stochastic processes as well. We review the few ad hoc network processes with dependent nodes studied to date and show how they fit into our general framework.

The other topics in the paper are as follows:

**Blocking of Certain transitions Due to Network Constraints**. We show how to model blocking when the network process is not entirely reversible - prior results required reversibility or used approximations.

**Palm Probabilities of a Network at its Transitions**. A remarkable property of an open Jackson network is that when a unit moves from one node to another, the probability distribution of the unmoved units is the same as that for the entire network process. That is, a "moving unit sees time averages". We show that this MUSTA property is really an application of Palm probabilities, which are essentially conditional probabilities conditioned on events of zero probability. This was apparently not known before - several papers are now starting to appear on this topic. We describe general Palm probabilities for a variety of transitions and give necessary and

sufficient conditions for the MUSTA property.

**Poisson Flows in Networks.** The flows of units between two nodes or the departures from a network process are sometimes Poisson processes. Knowing that the departure flow from a network is Poisson is useful when the flow enters another system such as a post- processor or inventory system and one wants to describe the behavior of the auxiliary system. We give easily checkable necessary and sufficient conditions for flows to be Poisson and characterize when they might be independent.

In summary, the major contribution in this paper is the discovery of the new multivariate distribution for representing the equilibrium behavior of Markovian networks. This appears to be a major building block in the theory of network processes with dependent nodes and routings.

# 3 Partially Balanced Markovian Processes

Much of the work on this topic and the topics in the remaining sections are documented in the Ph.D dissertation by Kwangho Kook titled *Equilibrium Behavior of Markovian Network Processes*, June 1989. We will prepare several papers on these topics this summer.

All of the Markovian network processes studied to date that have known closed-form expressions for their equilibrium distribution satisfy a certain "partial balance" property. Our work described above covers a large subclass of these partially balanced processes, but the entire class of partially balanced processes that have not been characterized is vast in comparison. We initially thought that researchers would develop the equilibrium theory for partially balanced networks by identifying and studying isolated subclasses of these processes. A universal equilibrium distribution for all such processes seemed unlikely. We have found, however, that a universal theory is indeed possible and we have developed much of it only recently. The generic problems we are addressing are:

**(a)** Find a general form for the equilibrium distribution of partially balanced networks.

**(b)** Find necessary and sufficient conditions on the transition rates of the process for it to be partially balanced.

We have essentially solved these problems for Markovian networks in which units move one at a time. We have also begun to solve them for networks with concurrent movements of units- little is known about these networks, which are described later. This unraveling of the mystery of partially balanced networks is a major breakthrough in the understanding of dependencies in networks. We are extremely pleased with this result.

# 4 Passage Times in Networks

A long-standing problem in stochastic networks, even for Jackson networks, is to find the mean passage time for a unit to move from one sector of a network to another sector. We have solved this problem for very general processes. This also led us to the study of mean passage times for a variety of routes in a network. The difficulty in this topic is that one cannot approach the problem by standard Markovian reasoning. When a unit begins a passage on a route, it is not known whether the unit will complete the route until the unit reaches the end of the route. In other words, the numbers of units undergoing a passage at any time is a function of the future of the process as well as its past. We overcome this difficulty by a subtle labeling device that allows us to look into the future in a certain regenerative sense. Our expressions for mean passage times provide new performance parameters for assessing the quality of a network.

# 5 Networks with Concurrent Movements

In actual networks, batch processing and splittng and merging of units are more common than not. These are examples of what we call concurrent movement of units. Little is known about networks with concurrent movements. We have begun to study these networks along the same lines as discussed above.

An important type of concurrent movement is the modeling of resource sharing in a network where the processing at a node requires the use of an auxiliary resource, e.g. computer file, machine tools, pallets. The resources can be represented as artificial units and their normal storage areas as artificial nodes. When a usual unit enters a node for processing, the

artificial units also move simultaneously to the node. When the processing is complete, all the units depart simultaneously. This problem area of resume sharing in networks is relevant to many types of real world problems in manufacturing and computer systems.

# 6 Service Stations with Batch Arrivals and Batch Services

As a first step in developing the theory of networks with concurrent movements, we studied several typical processing rules for one node in isolation. We eventually developed two models for a service center with batch arrivals and batch services. These models are important in their own right as well as in a network context.

The first model is an $M_n^b/M_n^b/1$ system. This is a single-server system in which batches of units arrive according to a Poisson process with rate depending on the number in the system and the batch sizes are i.i.d. and have a geometric distribution. The neumonic $M_n^b$ refers to this type of process. Similarly, the service process, also represented $M_n^b$, is a batch service system in which batches depart according to a Poisson process with rate depending on the number in the system and the batch sizes are independent truncated geometric variables. We derive the equilibrium distribution of the number of units in this $M_n^b/M_n^b/1$ system.

One can also interpret the $M_n^b/M_n^b/1$ system as a generalized birth and death process where the births and deaths occur in batches or groups. Our results are therefore applicable in settings where traditional birth and death processes have been used. A special case is the classical $M^b/M/1$ queueing system ($M^b$ means compound Poisson with geometric batches). Other special cases of the $M_n^b/M_n^b/1$ system, which have not been studied before, are the systems $M_n^b/M/s, M_n^b/M/\infty$, $M_n^b/M_n/1, M_n/M_n^b/1$, etc. ($M_n$ means state-dependent Poisson Process).

The second model we study in this chapter is an $M^b/M^B/1$ system. This is a single server system in which batches of units arrive according to a Poisson process and the batch sizes are i.i.d. geometric variables. The service process is a batch service system in which the $B$ units are served together, except when less than $B$ units are in the system and ready for

service, at which time all units are served. The service time for a batch is exponentially distributed. We derive the equilibrium distribution of the number of units in this system. In our future research, networks with concurrent movements where the nodes operate like these batch service systems.

# 7 Appendix

Cumulative List of Papers During Grant Period 9/84 to 4/89.

All but the last one are authored by R. F. Serfozo.


Partitions of point processes: multivariate Poisson approximations, *Stoch. Processes Appl.* 20, 281-294, 1985.

Compound Poisson approximations for sums of random variables, *Ann. Probability* 14, 1391-1398, 1986.

Heredity of stationary and reversible stochastic processes, *Adv. Appl. Probability* 18, 574-576, 1986.

Equitable transit charges in multi-administration telecommunications networks, *Queueing Systems: Theory and Applications* 2, 83-92, 1987.

Point Processes. Technical Report, Center for Stochastic Processes, University of North Carolina, 1987. Part of ORSA handbook on stochastic processes.

Extreme values of birth and death processes and queues, *Stoch. Processes Appl.* 27, 291-306, 1988.

Extreme values of queue lengths in M/G/1 and G/M/1 systems, *Math. Oper. Res.* 13, 349-357, 1988.

Poisson functionals of Markov processes and queueing networks, *Adv. Appl. Probability* (to appear) 1989.

Markovian network processes: congestion-dependent routing and processing, *Queueing Systems* (to appear) 1989.

Networks of queues with blocking and load balancing, joint with K.E. Chin, *Proceedings of Material Handling Forum* (to appear) 1989.

Equilibrium Behavior of Markovian Network Processes, by Kwangho Kook. Ph.D. Dissertation in Industrial and Systems Engineering, Georgia Institute of Technology. 1989.